Outage Analysis and Optimal Power allocation for Network-coding-based Hybrid AF and DF

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Abstract—Network coding was proposed to increase the achievable throughput of multicast in a network. Recently, combining network coding into user cooperation has attracted research attention. For cooperative transmission schemes with network coding, users combine their own and their partners messages by network coding. In previous works, it was shown that adaptive DF with network coding can achieve diversity gain and additional throughput gain. In this paper, to improve performance of conventional protocols and maximize advantage of using network coding, we propose a new network coding based user cooperation scheme which uses adaptively amplify-and-forward and decode-and-forward according to inter-user channel status. We derive outage probability bound of proposed scheme and prove that it has full diversity order in the high SNR regime. Moreover, based on the outage bound, we compute optimal power allocation for the proposed scheme.

I. INTRODUCTION

Relay-assisted transmission schemes for wireless networks are continuing to grow because of their potential of providing the benefits of space diversity without a need for physical antenna arrays. As an extension of relay-assisted system, user-cooperative transmission scheme was proposed in [1] and transmission algorithm of relay-assisted system including user cooperation can be divided into two categories: amplify-and-forward (AF) and decode-and-forward (DF). The AF scheme transmits received data after amplifying it, and it provides good performance with reliable source-relay channel condition. The DF scheme transmits received data after decoding and re-encoding it. To improve the conventional DF scheme, Adaptive decode-and-forward scheme (ADF) is proposed in [2], it is modified DF scheme which the source (=User 1) transmits their information to the destination again, when the relay fails to decode. [3] proposed channel coded user cooperation providing coding gain as well as path diversity gain.

Network coding (NC) to increases throughput of multicast network was firstly proposed by Yeung et. al. in [4]. Combining the network coding concept with user cooperation was introduced in recent papers [5][6]. [5] applied NC to user cooperation and distributed antenna systems using a DF scheme on a perfect user-partner channel assumption. Applying NC to cooperative transmission schemes, users combine their own and their partner’s message by network coding operation (i.e., modulo-2 addition). They showed that NC can give additional diversity gain and throughput gain with low complexity operations. [6] suggested network-coding- based adaptive DF (NC ADF) over unreliable inter-user channel. It used the DF scheme only when a user decoded partner’s data successfully. On the other hand, due to limited battery lifetimes of the small devices in emerging wireless applications, e.g., sensor networks, optimal power allocation schemes for various cooperative systems have been investigated. Especially, optimal power allocation scheme that minimizes the obtained outage probability bound under a total power constraint was proposed in [7][8].

In this paper, for improving performance of conventional protocols and maximizing advantage of using network coding, we apply hybrid relaying to network-coding-based protocol (NC HAD). Hybrid relaying was proposed in [9], and it divided all relays into DF and AF relay groups according to success and failure of decoding. We show that proposed scheme outperforms conventional protocols on outage probability and prove that it has full diversity at relatively high-SNR regime. Moreover, based on the derived outage probability bound, we investigate optimal power allocation for NC HAD, assumptions on the availability of channel state information (CSI)

II. SYSTEM MODEL AND PREVIOUS WORKS

A. System model

We consider a half-duplex dual-hop communication scenario with orthogonal channel, where there are two users and one destination (base station). The information signal from the user is transmitted via partner as well as the direct path from the user to destination. During the first hop, each user broadcasts its information to the destination and the partner through orthogonal channel. The received signals at the destination $r_{1,d}$ and User 2 (partner) $r_{1,2}$ can be written as
\[ r_{1,d} = \sqrt{P_1 h_{1,d}} x + n_{1,d} \]
\[ r_{1,2} = \sqrt{P_1 h_{1,2}} x + n_{1,2} \]

where \( P_1 \) is the User 1’s transmitted power, \( n_{1,d} \) and \( n_{1,2} \) is the Additive Gaussian noise (AWGN) at the destination and the partner, respectively. The noise terms are modeled as zero-mean complex Gaussian random variables with variance \( N_0/2 \) per dimension. And, \( h_{1,d} \) and \( h_{1,2} \) are the channel coefficients from the User 1 and destination via User 2. The received signal at the destination in phase 2 is given by

\[ r_{2,d} = \beta_i h_{2,d} r_{1,2} + n_{2,d} \]

where \( \beta_i \) can be different to the User 2’s transmission protocol. If the User 2 transmits data using AF protocol, \( \beta_i \) satisfies the power constraints \( \beta_i \leq P_2 / (P_1 [h_{1,2}]^2 + N_0) \), where \( P_2 \) is the User 2’s transmitted power. Reversely, if the User 2 transmits data using DF protocol, \( \beta_i \) is \( \sqrt{P_1} \). The channel coefficient \( h_{1,2} \), \( h_{1,d} \) and \( h_{2,d} \) are the modeled as zero-mean complex Gaussian random variables with variance \( \delta_{1,2}^2 \), \( \delta_{1,d}^2 \) and \( \delta_{2,d}^2 \) respectively.

The maximum average mutual information can be represented as \( I_D = \log_2 (1 + \frac{P_1 [h_{1,d}]^2}{N_0}) \) (b/s/Hz). Outage event is defined as the case that the capacity falls below the threshold rate \( R \), such as \( I_D < R \), equivalently \( [h_{1,d}]^2 < (\frac{2^R - 1}{P_1}) N_0 \).

For Rayleigh fading, \( [h_{1,d}]^2 \) exponentially distributed with parameter \( \delta_{1,d}^2 \). Then, outage probability for Rayleigh fading channel can be evaluated as

\[ P_{out} = \Pr (I_D < R) = \Pr \left( [h_{1,d}]^2 < \frac{(2^R - 1) N_0}{P_1} \right) \]
\[ = 1 - \exp \left( -\frac{(2^R - 1) N_0}{P_1 \delta_{1,d}^2} \right) - \frac{(2^R - 1) N_0}{P_1 \delta_{1,d}^2} \approx \frac{(2^R - 1) N_0}{P_1 \delta_{1,d}^2} \]

where \( \Gamma \) is received mean SNR of fading channel and we use approximate method to compute outage bound, that is, “\( f(x) \approx g(x) \) when \( x \to 0 \)” indicates “\( \lim_{x \to 0} \frac{f(x)}{g(x)} = 1 \)” where both \( f(x) \) and \( g(x) \) are function of \( x \).

### III. NETWORK-CODING-BASED HYBRID RELAYING

#### A. Outage probability of Network-coding-based Hybrid AF and DF

In this section, we explain frame structure and derive outage probability of NC HAD. Data transmission consists of two frames as in the previous protocols. This result can be easily expanded for \( n \) users with \( 2^n \) cases. Figure 1 shows the

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**Fig. 1** Frame structure of NC HAD. Where the \( \times \) denotes that decoding fail at the node.
frame structure of NC HAD for the four cases. We analyze the conditional channel capacity and outage event based on the Maximum ratio combining (MRC). Each case is denoted by $\theta \in \{1,2,3,4\}$

- **Case 1**: This is identical to Case 1 of NC ADF in [5]. At the first frame, both users successfully decode each other. In an information-theoretic sense, correct decoding corresponds to

$$C^1_{1,2}(\gamma_{1,2}) = \frac{1}{2} \log (1 + \gamma_{1,2}) > R \quad (4)$$

$$C^1_{2,1}(\gamma_{2,1}) = \frac{1}{2} \log (1 + \gamma_{2,1}) > R$$

where the subscript of $\gamma_{1,2}$ denotes instantaneous SNR of transmitting User 1 to User 2 and superscript $k$ of $C^1_{ij}$ represents the frame which the user transmits. We only consider the outage probability of User 1, because of the symmetric property between two users. If both transmitted data in the first frame from User 1 and User 2 are not decoded successfully, it is an outage regardless of the remaining transmissions and it can be written as

$$\{C^1_{1,2} < R\} \cdot \{C^1_{2,1} < R\} = \Pr\{\gamma_{1,2} < 2^{2R} - 1\} \cdot \Pr\{\gamma_{2,1} < 2^{2R} - 1\} \quad (5)$$

If the transmitted data in the first frame from User 1 are not decoded successfully and the transmitted data in the first frame from User 2 are decoded successfully, then the outage occurs only when network coded data in the second frame fails. It can be written as

$$\{C^1_{1,2} < R\} \cdot \{C^2_{1,2} > R\} \cdot \{C^2_{2,1} > R\} \cdot \{C^2_{1,2} < R\} = \Pr\{\gamma_{1,2} < 2^{2R} - 1\} \cdot \Pr\{\gamma_{2,1} < 2^{2R} - 1\} \cdot \Pr\{\gamma_{1,2} > 2^{2R} - 1\} \cdot \Pr\{\gamma_{2,1} > 2^{2R} - 1\} \quad (6)$$

As a result, the overall outage probability of Case 1 for User 1 can be obtained as

$$P_{out}^{Case 1} = \Pr\{\gamma_{1,2} > 2^{2R} - 1\} \cdot \Pr\{\gamma_{2,1} > 2^{2R} - 1\} \cdot \Pr\{\gamma_{1,2} < 2^{2R} - 1\} \cdot \Pr\{\gamma_{2,1} < 2^{2R} - 1\}$$

- **Case 2**: All of the transmitted data from each user in the first frame are in outage. Instead of transmitting their own bits, User 1 transmits User 2’s data bit and User 2 transmits User 1’s data using the AF scheme. In this case, using hybrid AF and DF cooperation, there is additional path diversity gain from transmitting the partner’s data bit through its own channel. The overall outage probability of Case 2 can be shown as

$$P_{out}^{Case 2} = \Pr\{\gamma_{1,2} < 2^{2R} - 1\} \cdot \Pr\{\gamma_{2,1} < 2^{2R} - 1\}$$

$$\cdot \Pr\{\gamma_{1,2} > 2^{2R} - 1\} \cdot \Pr\{\gamma_{2,1} > 2^{2R} - 1\} \quad (8)$$

We define new variable $\gamma'_{AF}$ such that

$$\gamma'^1_{AF} = \frac{\gamma_{2,1,2,d} \gamma_{1,1,d} + \gamma_{1,2,1,d} \gamma_{2,2,d}}{\gamma_{1,2} + \gamma_{2,2,d} + 1}$$

where subscript $r$ denotes who relays user’s data using AF.

- **Case 3**: The first frame of Case 3 is the same as the NC ADF and NC HAD. That is, User 1 fails to decode and User 2 decodes successfully. Thus, User 1 forwards received data using the AF scheme, whilst User 2 forwards modulo-2 operated data of User 1 and User 2. In order to obtain outage probability of User 1, $C^1_{1,2}$ should be outage as in Case 1. If $C^1_{1,2}(\gamma'_{AF} | \theta = 3)$ is more than $R$, User 1 is in outage, regardless whether or not decoding successfully in the second frame. Conversely, if the AF data of User 2 is decoded successfully, outage occurs when the transmitted message of the second frame cannot be decoded at the base station. The overall outage probability of Case 3 can be written

$$P_{out}^{Case 3} = \Pr\{\gamma_{1,2} > 2^{2R} - 1\} \cdot \Pr\{\gamma_{2,1} < 2^{2R} - 1\}$$

$$\cdot \Pr\{\gamma_{1,2} < 2^{2R} - 1\} \cdot \Pr\{\gamma_{2,1} < 2^{2R} - 1\} \cdot \Pr\{\gamma_{1,2} > 2^{2R} - 1\} \cdot \Pr\{\gamma_{2,1} > 2^{2R} - 1\} \quad (9)$$

- **Case 4**: Case 4 is identical to Case 3 with the roles of User 1 and 2 reversed. The overall outage probability of Case 4 can be written as

$$P_{out}^{Case 4} = \Pr\{\gamma_{1,2} < 2^{2R} - 1\} \cdot \Pr\{\gamma_{2,1} > 2^{2R} - 1\}$$

$$\cdot \Pr\{\gamma_{1,2} > 2^{2R} - 1\} \cdot \Pr\{\gamma_{2,1} > 2^{2R} - 1\} \quad (10)$$

For the independent inter-user channel, the overall outage probability of User 1 can be written the sum of each case as

$$P_{out} = P_{out}^{Case 1} + P_{out}^{Case 2} + P_{out}^{Case 3} + P_{out}^{Case 4} \quad (11)$$

For the reciprocal inter-user case (i.e., $\gamma_{1,2} = \gamma_{2,1}$), only Case 1 and 2 can be considered. Therefore, (11) can be simplified to

$$P_{out} = \Pr\{\gamma_{1,2} > 2^{2R} - 1\} \cdot \Pr\{\gamma_{1,2} < 2^{2R} - 1\}$$

$$\cdot \Pr\{\gamma_{1,2} > 2^{2R} - 1\} \cdot \Pr\{\gamma_{1,2} > 2^{2R} - 1\} \quad (12)$$

Then, using (3), overall the outage bound of NC HAD in the reciprocal inter-user case can be written as
\[ P_{\text{out,HAD}}^{NC} = \left( 1 - \frac{(2^R - 1)N_0}{P_1 \delta_{1,2}^2} \right) \left( \frac{(2^R - 1)N_0}{P_1 \delta_{1,2}^2} \right) + \left( \frac{(2^R - 1)N_0}{P_2 \delta_{2,2}^2} \right) \left( \frac{(2^R - 1)N_0}{2P_1 \delta_{1,2}^2 P_2 \delta_{2,2}^2} \right) \]

\[ \left( \frac{(2^R - 1)N_0}{P_1 \delta_{1,2}^2} \right) + \frac{(2^R - 1)N_0}{2P_1 \delta_{1,2}^2 P_2 \delta_{2,2}^2} \]

(13)

B. Optimal power allocation for NC HAD

In this section, we assume that each node knows all CSI by using appropriate channel estimation and feedback schemes. We can then adaptively allocate the transmit powers used by User 1 and partner, subject to the constraint that the total power \( P \). The optimal power allocation is based on minimizing the outage probability bound in (13) under total power constraint. Removing the fixed terms from outage probability bound, the optimization problem can be written as

\[ \mathbf{p}_{opt} = \arg \min_p \left( \frac{P_1 \delta_{1,2}^2 - K}{P_1 P_2} + \frac{P_1 \delta_{1,2}^2 + P_2 \delta_{2,2}^2}{P_1 P_2} \right) \]

subject to

\[ P_1 + P_2 \leq P, \quad P_i \geq 0, \quad \forall i \]

where \( K = (2^R - 1)N_0 \) and \( \mathbf{p} = [P_1, P_2] \) is the vector of the power. The Lagrangian of this optimization problem can be written as

\[ L = \frac{P_1 \delta_{1,2}^2 - K}{P_1 P_2} + \frac{P_1 \delta_{1,2}^2 + P_2 \delta_{2,2}^2}{P_1 P_2} + \lambda (P_1 + P_2 - P) + \sum_{i=1}^2 \mu_i (P_i - P_i) \]

where the \( \mu_i \)'s serve as the slack variables. To minimize the outage bound, it is clear that we must have \( P_i > 0 \) \( \forall i \). The complementary slackness imply that since \( P_i > 0 \), then \( \mu_i > 0 \) \( \forall i \). Knowing that the log function is a monotone increasing function and defining the vector \( \mathbf{a} = [a_1, a_2, \ldots] \), where \( a_1 = P_1 / P \) and \( a_2 = P_2 / P \), the Lagrangian of the optimization problem in (15) can now be given as

\[ f = -2 \log a_1 - \log a_2 \]

\[ + \log \left( (a_1 \delta_{1,2}^2 - K) \left( 1 + \frac{1}{a_1} + \delta_{1,2}^2 + \frac{P_2}{P_1} \delta_{2,2}^2 \right) \right) + \lambda (\mathbf{a}^T \mathbf{1} - 1) \]

where \( \mathbf{I}_{N+1} \) is an all 1 \((N+1) \times 1 \) vector. Applying first-order optimal conditions, \( a_{opt} \) must satisfy

\[ \frac{\partial f}{\partial a_x} = \frac{\partial f}{\partial a_y} = 0 \]

(17)

Using (17), we can get

\[ \delta_{1,2}^2 a_1 - 4 \delta_{1,2}^2 + 3K + \frac{2K}{a_1} - 3 \delta_{2,2} a_2 \]

\[ = - \frac{a_1}{a_2} \delta_{1,2}^2 a_1 - 2 \delta_{1,2}^2 a_1 + K \frac{a_1}{a_2} + \frac{K}{a_2} \]

(18)

Define \( c_r = (a_r / a_s) = (P_r / P_s) \) and using (18), we can obtain

\[ c_{opt} = \left( \frac{3}{2} \delta_{2,2}^2 \right) \]

(19)

C. Diversity order of NC HAD

Diversity order of NC HAD is derived over Rayleigh fading channel using approximation method in [11]. We change outage bound (13) to equation which consists of mean SNR using (3), and re-parameterize the mean SNR \( \Gamma_{i,j} \) followings as

\[ \Gamma_{i,j} \rightarrow \Gamma_{i,j} \cdot \Gamma_i \]

where now \( \Gamma_i \) is the ratio of the user transmit power to the received noise, and \( \Gamma_{i,j} \) is a finite constant accounting for large scale path loss and shadowing effects. For the compute diversity order, we assume that \( \Gamma_i \) of both users is same. Relative influence of each channel are still captured by the \( \Gamma_{i,j} \).

This re-parameterization can be expressed outage probability as a function of \( 1 / \Gamma_i \), and then letting \( \Gamma_i \rightarrow \infty \) (e.g., the high-SNR regime), the diversity order is given by the smallest exponent of \( 1 / \Gamma_i \). First, we refer to [11], which means decoding successfully

\[ \Pr[\gamma_{ij} > 2^{2R} - 1] = \frac{2^R - 1}{\Gamma_i \Gamma_{ij}} + \frac{(2^R - 1)^2}{\Gamma_i^2 \Gamma_{ij}^2} + O(\frac{1}{\Gamma_i}) \]

(20)

And, we also use AF scheme of high SNR regime in [2]

\[ \Pr[\gamma_{ij} < 2^{2R} - 1] = \frac{(\Gamma_{i,1} + \Gamma_{2,2})(2^R - 1)^2}{2 \Gamma_i^2 \Gamma_{i,d} \Gamma_{2,d}^2} \]

(21)

To compute overall diversity order, we re-parameterize DF scheme of Case I as
\[
\Pr\{\gamma_{1,d} + \gamma_{2,d} < 2^R - 1\} = 2^{2^R-1} \int_0^1 \frac{1}{\Gamma_{1,d}} \exp(-\frac{\gamma_{1,d}}{\Gamma_{1,d}}) \frac{1}{\Gamma_{2,d}} \exp(-\frac{\gamma_{2,d}}{\Gamma_{2,d}}) d\gamma_{2,d} - \frac{k}{\Gamma_{1,d}} d\gamma_{2,d} \\
= \left[1 - \exp\left(\frac{1 - 2^R}{\Gamma_{2,d}}\right)\right] \int_0^{2^R-1} \frac{1}{\Gamma_{2,d}} \exp(-\frac{\gamma_{2,d}}{\Gamma_{2,d}}) - \frac{k}{\Gamma_{1,d}} d\gamma_{2,d} + O\left(\frac{1}{\Gamma_T^3}\right)
\]

where \( k = 2^R - 1 - \gamma_{2,d} \). We can change the right side of (22) using re-parameterizing method as

\[
\frac{1}{\Gamma_{2,d}} \int_0^{2^R-1} \frac{k}{\Gamma_{1,d}} \left[1 - \exp\left(\frac{\gamma_{2,d}}{\Gamma_{2,d}}\right)\right] d\gamma_{2,d} + O\left(\frac{1}{\Gamma_T^3}\right)
\]

(23)

Overall DF scheme of \( NC \ HAD \), (23) can be

\[
\Pr\{\gamma_{1,d} + \gamma_{2,d} < 2^R - 1\} = \frac{(2^R-1)^2}{2\Gamma_1^2\Gamma_2^2} + \frac{(2^R-1)^2}{2\Gamma_2^2\Gamma_{1,d}^2} + O\left(\frac{1}{\Gamma_T^3}\right)
\]

(24)

As a result, the diversity order of \( NC \ HAD \) is

\[
P_{out,1} = \frac{1}{\Gamma_T^2} \left[ \frac{(2^R-1)^2}{2\Gamma_1^2\Gamma_2^2} - \frac{(2^R-1)^2}{2\Gamma_2^2\Gamma_{1,d}^2} \right] + O\left(\frac{1}{\Gamma_T^3}\right)
\]

(25)

It is noted that proposed scheme has full diversity order with two users. This result can be easily extended to \( n \) users. The diversity order for the independent inter-user case is the same as that of the reciprocal inter-user case.

IV. NUMERICAL RESULTS

In this section, we present numerical results of proposed scheme \( NC \ HAD \). First, we prove our proposed protocol has better performance than existing protocols in the figure 2 and 3 for the equal power allocation, User 1’s power and partner’s power are in the same ratio, as natural choice. It compare with \( NC \ ADF \) for both reciprocal and independent inter-user channels over Rayleigh fading channel. Figure 2 presents outage probability for various uplink (user-base station) mean SNR with fixed inter-user mean SNR of 5dB. It can be noted that for both reciprocal and independent inter-user cases, the proposed protocol has lower outage probability than \( NC \ ADF \). The performance gain comes from additional path diversity. The performance gap between \( NC \ ADF \) and \( NC \ HAD \) for the reciprocal inter-user case is larger than that for the independent case. In order to show the path diversity gain in details, figure 3 compares outage probabilities of \( NC \ ADF \) and \( NC \ HAD \) for four different cases. There is no difference in Case 1 for both algorithm and performance. In Case 2 and 4 the path diversity gains of \( NC \ HAD \) are realizable. In Table I, we can get numerical results for the optimal power allocation for the various channel variance. From the results, it is clear that equal power allocation is not optimal. Figure 4 shows results of the optimal power allocation, it shows the analytical outage bound of \( NC \ HAD \) versus total transmitted power. We present \( NC \ HAD \) for equal power and optimal power allocation scheme with different channel variance \((\delta_{1,2}^2 = 3, \delta_{2,d}^2 = 1 \text{ and } \delta_{1,2}^2 = 1, \delta_{2,d}^2 = 5)\). Since User2 data
decode partner’s successfully and each user transmits same data (ex-or) in second frame in high inter-user channel variance, outage performance is determined by total transmit power not User 1 or User 2’s individual power. Hence, when User 1 to User 2’s channel variance is high (e.g., $\delta_{1,2}^2 = 3$, $\delta_{2,4}^2 = 1$), optimal power allocation of NC HAD is same as equal power allocation. Reversely, if channel variance between relay to destination is high, the outage is affected by each user’s power rate because each case (Case 1 and Case 2) occurs independently and it can result in some performance gain by optimal power allocation.

V. CONCLUSION

In this paper, for improving performance of conventional protocols and maximizing advantage of using network coding, we apply hybrid relaying to network-coding-based protocol, called Network coding based Hybrid AF and DF (NC HAD). We derive outage probability bound of proposed scheme and show that it has better performance than previous schemes in analytical and numerical results. We also prove that NC HAD has full diversity in high SNR regime and optimal power allocation can achieve additional performance gain.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>OPTIMAL POWER ALLOCATION FOR NC HAD (FIXED $\delta_{1,2}^2 = 1$)</th>
</tr>
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<tbody>
<tr>
<td>Power ratio of each node</td>
<td>$P_1 / P = 0.5$, $P_2 / P = 0.5$</td>
</tr>
<tr>
<td>$\delta_{1,2}^2 = 3$, $\delta_{2,4}^2 = 1$</td>
<td></td>
</tr>
<tr>
<td>$\delta_{1,2}^2 = 1$, $\delta_{2,4}^2 = 1$</td>
<td>$P_1 / P = 0.7$, $P_2 / P = 0.7$</td>
</tr>
<tr>
<td>$\delta_{1,2}^2 = 1$, $\delta_{2,4}^2 = 5$</td>
<td>$P_1 / P = 0.6818$, $P_2 / P = 0.3182$</td>
</tr>
<tr>
<td>$\delta_{1,2}^2 = 1$, $\delta_{2,4}^2 = 8$</td>
<td>$P_1 / P = 0.7742$, $P_2 / P = 0.2258$</td>
</tr>
</tbody>
</table>

Outage probability of Hybrid NC HAD for equal power and optimal power allocation scheme.

REFERENCES